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Neutrino Flavor Mixing in the 'Doublet-Singlet Oscillation' Model

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Abstract

Recently, it was pointed out that the solar and atmospheric neutrino deficits can be explained by oscillations between electroweak doublet and singlet neutrinos in the model of Majorana neutrinos. However, since the model includes no flavor mixing, it cannot explain the recent LSND result. We extend the model to include the flavor mixing, and obtain the explanation of the LSND result together with the solar and atmospheric neutrino deficits. The requirement for the neutrinos to be the hot dark matter selects out only the vacuum oscillation solution to the solar neutrino deficit.

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The recent LSND experiment [1] may be the first observation of the neutrino oscillation in $\overline{\nu_\mu} \to \overline{\nu_e}$ conversion. The experiment obtained values of parameters of the oscillation as $\Delta m^2_{_{LSND}} \simeq (0.25\text{–}2.5)\ \mathrm{eV}^2$ and $\sin^2 2\theta_{_{LSND}} \simeq 2\times 10^{-3} - 4\times 10^{-2}$. However, the LSND data is still somewhat controversial: in ref.[2] it is interpreted as an upper bound on the parameters of the neutrino oscillation. In this latter, we accept the data as the evidence of the neutrino oscillation. Then, we should consider the LSND result together with another neutrino oscillation phenomena, i.e. the solar neutrino deficit [3] and the atmospheric neutrino anomaly [4].

Recently, it was pointed out that the solar and atmospheric neutrino deficits can be explained by oscillations between electroweak doublet and singlet neutrinos in the model of the Majorana neutrino without flavor mixing [5]. In the following, this type of neutrino oscillation is called 'doublet-singlet oscillation'. The solar electron neutrino deficits can be explained by the 'doublet-singlet oscillation' in the first generation (two solutions are possible for the values of $\Delta m^2_\odot$ and $\sin^2 2\theta_\odot$ as they will be mentioned in the following), and the atmospheric muon neutrino anomaly, which requires $\Delta m^2_\oplus \simeq 10^{-2}\ \mathrm{eV}^2$ and $\sin^2 2\theta_\oplus \simeq 1$, can be done by the 'doublet-singlet oscillation' in the second generation. Furthermore, neutrinos can be both two components of the dark matter in the cold plus hot dark matter cosmological models [6]. While some neutrinos in the first and second generation can be the hot dark matter, tau neutrino can be the cold dark matter. There are two kinds of solutions to the hot dark matter neutrino. One is that only two neutrinos in the second generation are the hot dark matter, if we choose a solution called the 'small-angle MSW solution' [7] ($\Delta m^2_\odot \simeq 10^{-5}\ \mathrm{eV}^2$ and $\sin^2 2\theta_\odot \simeq 10^{-2}$) in the first generation. The other is that all of the neutrinos in two generations are the hot dark matter with the choice of the vacuum oscillation solution [8] ($\Delta m^2_\odot \simeq 10^{-10}\ \mathrm{eV}^2$ and $\sin^2 2\theta_\odot \simeq 1$).

However, it is clear that this model cannot explain the $\overline{\nu_e}$ appearance from $\overline{\nu_\mu}$ in the LSND experiment, since there is no flavor mixing. In this latter, we extend the model in

ref. [5] to include flavor mixing between the first and second generations [1]. We will show that this extension selects only one out of two solutions to the hot dark matter neutrino.

We extend the standard model by introducing three right-handed neutrinos and one electroweak singlet scalar Φ [10]. The Yukawa couplings are described by

$$\mathcal{L}_{\text{Yukawa}} = -\overline{\nu_L}(g_{_Y}\phi)\nu_R - \overline{\nu_R{}^c}(g_{_M}\Phi)\nu_R + h.c. \ \ , \tag{1}$$

where ϕ is the electric-charge neutral component of the Higgs field in the standard model, $g_{_Y}$ and $g_{_M}$ are 3×3 matrices, and $\nu_{L,R}$ are column vectors. The Dirac and Majorana mass terms appear by the non-zero vacuum expectation values of above scalar fields. To our aim, we focus the first and second generations. Then the mass matrix is given by

$$\left[\begin{array}{cc} 0 & m_{_D} \\ m_{_D}^T & M \end{array}\right] , \tag{2}$$

where $m_{_D} = g_{_Y}\langle\phi\rangle$ is the 2×2 Dirac mass matrix, and $M = g_{_M}\langle\Phi\rangle$ is the 2×2 Majorana mass matrix. In order to discuss neutrino oscillations, we should diagonalize the 4×4 mass matrix in eq.(2). This diagonalization is complicated in general. However, to our aim, it is satisfactory to consider a simple case. We assume that two matrices $m_{_D}$ and M can be simultaneously diagonalized: $U_L^\dagger m_{_D} U_R = diag[m_1^{_D}, m_2^{_D}]$ and $U_R^T M U_R = diag[M_1, M_2]$, where U_L and U_R are the 2×2 orthogonal matrix [2] for rotations of left and right-handed states, respectively.

Without loss of generality, we start from $U_R = 1$. Denoting the left-handed flavor eigenstate and mass eigenstate as $\Psi_f = (\nu_{eL}, \nu_{\mu L}, \nu_{eR}{}^c, \nu_{\mu R}{}^c)^T$ and $\Psi_m = (\nu_1, \nu_2, \nu_3, \nu_4)^T$, respectively, the mixing matrix is given by

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[1] Our model is the same, in form, as the models in ref. [9], which treat four neutrinos; three 'active' neutrinos, ν_e, ν_μ and ν_τ, and one 'sterile' neutrino.

[2] In this letter, we neglect CP violation phases.

$$\Psi_m = \begin{bmatrix} U_L & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} C_\odot & 0 & S_\odot & 0 \\ 0 & C_\oplus & 0 & S_\oplus \\ -S_\odot & 0 & C_\odot & 0 \\ 0 & -S_\oplus & 0 & C_\oplus \end{bmatrix} \Psi_f \ , \tag{3}$$

where $C_{\odot,\oplus}$ and $S_{\odot,\oplus}$ denote $\cos\theta_{\odot,\oplus}$ and $\sin\theta_{\odot,\oplus}$, respectively. Note that the model in ref. [5] appear as the limit $U_L \to 1$. In this case, the solar and atmospheric neutrino deficits are explained with $\nu_{eL} \to {\nu_{eR}}^c$ and $\nu_{\mu L} \to {\nu_{\mu R}}^c$ conversions, respectively. The 2×2 orthogonal matrix U_L is described by

$$U_L = \begin{bmatrix} C_{_{LSND}} & S_{_{LSND}} \\ -S_{_{LSND}} & C_{_{LSND}} \end{bmatrix} \ , \tag{4}$$

where $C_{_{LSND}}$ and $S_{_{LSND}}$ denote $\cos\theta_{_{LSND}}$ and $\sin\theta_{_{LSND}}$, respectively. The matrix U_L corresponds to the flavor mixing between two electroweak doublet neutrinos.

Then, we calculate neutrino oscillation probabilities. Note that the flavor mixing angle required by the LSND experiment is small $S_{_{LSND}} \ll 1$, and the three mass square differences required by the solar and atmospheric neutrino deficits and the LSND experiment are hierarchical, namely, $\Delta m^2_\odot \ll \Delta m^2_\oplus \ll \Delta m^2_{_{LSND}}$. These features reduce our analysis of the neutrino oscillation from four flavor case to combinations of two flavor cases. We define three mass square differences as $\Delta m^2_\odot = m_3^2 - m_1^2$, $\Delta m^2_\oplus = m_4^2 - m_2^2$, and $\Delta m^2_{_{LSND}} = |m_1^2 - m_2^2|$, where m_i is the Majorana mass eigenvalue of ν_i $(i = 1, 2, 3, 4)$.

The probability of $\nu_\mu \to \nu_e$ conversion which corresponds to the LSND experiment is given by

$$\begin{aligned} P_{\nu_\mu \to \nu_e} &\simeq \sin^2 2\theta_{_{LSND}} \sin^2\left(\frac{\Delta m^2_{_{LSND}} L}{4E}\right) - \frac{1}{4}\sin^2 2\theta_{_{LSND}} \sin^2 2\theta_\odot \sin^2\left(\frac{\Delta m^2_\odot L}{4E}\right) \\ &\quad - \frac{1}{4}\sin^2 2\theta_{_{LSND}} \sin^2 2\theta_\oplus \sin^2\left(\frac{\Delta m^2_\oplus L}{4E}\right) \\ &\simeq \sin^2 2\theta_{_{LSND}} \sin^2\left(\frac{\Delta m^2_{_{LSND}} L}{4E}\right) \ . \end{aligned} \tag{5}$$

Here, we use $\sin(\Delta m^2_\odot L/4E) \ll 1$ and $\sin(\Delta m^2_\oplus L/4E) \ll 1$ at the energy range ($E =$ (36–60) MeV) and the oscillation length ($L = 30$m) in the LSND experiment and the hierarchy

among the three mass square differences.

The survival probability of $\nu_e \to \nu_e$ which corresponds to the solar neutrino deficit is given by

$$P_{\nu_e \to \nu_e} \simeq C^4_{_{LSND}} \left[1 - \sin^2 2\theta_\odot \sin^2 \left(\frac{\Delta m^2_\odot L}{4E} \right) \right] + S^4_{_{LSND}} \left[1 - \sin^2 2\theta_\oplus \sin^2 \left(\frac{\Delta m^2_\oplus L}{4E} \right) \right] \\ + \frac{1}{2} \sin^2 2\theta_{_{LSND}} \cos \left(\frac{\Delta m^2_{_{LSND}} L}{2E} \right) , \tag{6}$$

where the hierarchy among the three mass square differences is used. Since $S_{_{LSND}} \ll 1$, the second term in the right-hand side in eq.(6) can be ignored. Note that there is no resonance $\nu_e \to \nu_\mu$ conversion by the matter effect in the sun [7], because of the large value of $\Delta m^2_{_{LSND}}$. Thus, the third term can be also ignored, and we can reduce eq.(6) to

$$P_{\nu_e \to \nu_e} \simeq 1 - \sin^2 2\theta_\odot \sin^2 \left(\frac{\Delta m^2_\odot L}{4E} \right) = 1 - P_{\nu_e \to \nu_e^c} , \tag{7}$$

where we use $S_{_{LSND}} \ll 1$, and $P_{\nu_e \to \nu_e^c}$ is the conversion probability from ν_{eL} into its singlet partner ${\nu_{eR}}^c$.

Finally, the survival probability of $\nu_\mu \to \nu_\mu$ which corresponds to the atmospheric neutrino deficit is given by the exchange $\odot \leftrightarrow \oplus$ in eqs.(6) and (7), that is,

$$P_{\nu_\mu \to \nu_\mu} \simeq 1 - \sin^2 2\theta_\oplus \sin^2 \left(\frac{\Delta m^2_\oplus L}{4E} \right) = 1 - P_{\nu_\mu \to \nu_\mu^c} . \tag{8}$$

Here $S_{_{LSND}} \ll 1$ and the hierarchy are also used, and $P_{\nu_\mu \to {\nu_\mu}^c}$ is the conversion probability from $\nu_{\mu L}$ into its singlet partner ${\nu_{\mu R}}^c$. The flavor mixing due to U_L little affect the analysis of the solar and atmospheric neutrino deficits.

Let us discuss the mass spectrum of the neutrinos. Because of the hierarchy among the three mass square differences, $\Delta m^2_\odot \ll \Delta m^2_\oplus \ll \Delta m^2_{_{LSND}}$, only two types of mass spectrum are possible: type (i) $m_1 < m_3 < m_2 < m_4$ with $\Delta m^2_{_{LSND}} = m_2^2 - m_1^2$, type (ii) $m_2 < m_4 < m_1 < m_3$ with $\Delta m^2_{_{LSND}} = m_1^2 - m_2^2$.

The value of m_i is fixed by the solution to the hot dark matter neutrino. The sum of the neutrino masses is required $M_{HOT} = (5–7)$ eV in the cold plus hot dark matter models [6]. Then, we obtain

$$M_{HOT} = m_1 + m_2 + m_4 \simeq m_1 + 2\sqrt{m_1^2 \pm \Delta m^2_{LSND}}\ , \tag{9}$$

where the hierarchy $\Delta m^2_{\oplus} \ll \Delta m^2_{LSND}$ is used, and the signs + and − correspond to the type (i) and type (ii), respectively. Note that only one neutrino in the first generation contribute to M_{HOT}, since the electroweak singlet neutrino in the first generation has never been in thermal equilibrium in the thermal history of the universe (see ref. [11] for detailed discussion). From eq.(9), we can obtain the mass spectrum for the values of $\Delta m^2_{LSND} =$ (0.25–2.5) eV^2 and $M_{HOT} =$ (5–7) eV, which is shown in Tables I and II.

The mass spectrum, $m_1 \simeq m_3 \simeq O(\mathrm{eV})$, forces us to select the vacuum oscillation solution to the solar neutrino deficit. Let us discuss the mass matrix only in the first generation at the limit $U_L \to 1$. Because of $S_{LSND} \ll 1$, this limit is a good approximation. The mass matrix is given by

$$\begin{bmatrix} 0 & m_1^D \\ m_1^D & M_1 \end{bmatrix} . \tag{10}$$

The value of the mixing angle is related to the values of the matrix elements in eq.(10). The small mixing angle between doublet and singlet neutrinos requires the mass matrix to be the 'see-saw' type [12]: $m_1^D \ll M_1$. Then, we obtain $m_1 \simeq (m_1^D)^2/M_1 \ll M_1 \simeq m_3 \simeq \sqrt{\Delta m^2_{\odot}} \ll O(\mathrm{eV})$. Thus, the 'small-angle MSW solution' is disfavored. On the other hand, the large mixing angle, which corresponds to the vacuum oscillation solution, requires the mass matrix to be almost the 'pseudo-Dirac' type [13]: $m_1^D \gg M_1$. In this case, the value of m_1 can be treated as a free parameter, if the condition $m_1 \gg \Delta m^2_{\odot}$ is satisfied, and then we can fix $m_1 \simeq O(\mathrm{eV})$. Therefore, the vacuum oscillation solution to the solar neutrino deficit is selected out.

The value $m_1 \simeq m_3 \simeq O(\mathrm{eV})$ has no conflict with the experiments of the neutrinoless double beta decay [14], by which the effective electron neutrino mass is constrained as $\langle m_{\nu_e} \rangle < 0.68\mathrm{eV}$. Since we neglect CP violating phases, i.e. CP is conserved, two mass eigenstate, ν_1 and ν_3, have opposite CP eigenvalues ± 1, respectively. Thus, the cancelation occurs [13] in the calculation of the effective electron neutrino mass.

We extended the model of Majorana neutrino in ref. [5] to include the flavor mixing in the first and second generations. In this simple extended model, the LSND result can be explained by the oscillation between the electroweak doublet neutrinos, while the solar and atmospheric neutrino deficits can be done by the 'doublet-singlet oscillation' in the first and second generation, respectively, as well as in ref.[5]. The requirement for the neutrinos to be the hot dark matter selects out only the vacuum oscillation solution to the solar neutrino deficit.

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TABLES

TABLE I. The mass spectrum of type (i)

M_{HOT} (eV)	Δm^2_{LSND} (eV^2)	$m_1 \simeq m_3$ (eV)	$m_2 \simeq m_4$ (eV)
5	0.25	1.6	1.7
5	2.5	1.1	1.9
7	0.25	2.3	2.4
7	2.5	2.0	2.5

TABLE II. The mass spectrum of type (ii)

M_{HOT} (eV)	Δm^2_{LSND} (eV^2)	$m_1 \simeq m_3$ (eV)	$m_2 \simeq m_4$ (eV)
5	0.25	1.7	1.6
5	2.5	2.1	1.4
7	0.25	2.4	2.3
7	2.5	2.7	2.2